Course Syllabus (UQAM) Winter 2021

MAT993N : Hodge Theory, An introduction

Instructor : Steven Lu
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Office hour: Monday 4 :30-5 :30pm or by appointment

EVALUATION : homeworks 50% & exercices (with solutions)
two minor projects for exposé/presentation 50%

Prerequisite : some (multilinear) algebra, complex analysis, geometry and topology.

Abstract : The course focuses on a development of Hodge theory (over the complex numbers) starting with the Hodge structure of a complex curve and, more generally, of a Kähler variety. We will begin by giving the motivations and some historical backgrounds of Hodge theory that will quickly allow, some by analogy and skimming details, a selected overview of its modern manifestations. The emphasize, whenever expedient, will be on giving the foundational geometric, topological and analytical tools and perspectives that shaped the theory rather than detailing the well built up and documented algebraic theory. We will quickly develop the theory of Kähler and related varieties and their degenerations, referring to the current classic texts such as that of Voisin, of Arapura or of Huybrecht. Classical examples are then discussed by exposing their role in the classification of varieties and the study of their moduli spaces. We will also attempt a quick introduction to the theory of variations of Hodge structures and the relation to cycle spaces in order to have a view of the role of the intermediate jacobians and the Abel-Jacobi map that generalize the same for curves.

If time permits, we will discuss notions of stability, the Donaldson-Uhlenbeck-Yau correspondence and its generalizations, or introduce the rudiments of Carlos Simpson's non-abelian Hodge theory.

The first two weeks will focus on a motivational overview and thereafter we will cover quickly relevant parts of chapter II of the textbook by Arapura before starting Hodge theory proper following at least in spirit and the rough order given in the later parts of the texts of **Arapura and Voisin**.

PRINCIPLE REFERENCES (Books in the library) :
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1. Donu ARAPURA : Algebraic Geometry over the Complex Numbers

2. Claire VOISIN : Hodge Theory and Complex Algebraic Geometry

3. Daniel HUYBRECHTS : Complex Geometry

4. GRIFFITH-HARRIS : Principles of Algebraic Geometry, QA564G65.

Complex Analysis and Riemann Surface references : (pdfs by googling)
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a) W. Schlag : A concise course in complex analysis and Riemann surfaces

b) Otto Forester : Lectures on Riemann Surfaces QA 333 F6713

c) Armin Rainer : Introduction to Riemann Surfaces (following c))

d) Tim Perutz notes taken by Arun Debray : Riemann Surfaces

e) (Riemann surface w/ exercices) J-Y. Welschinger : Surfaces de Riemann.

References for projets and additional topics
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To be given by email.