MAT 7200: Homological Algebra/Algèbre Homologique

Hiver 2022

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Language of instruction: The language of instruction for this course will be English. Written work may be submitted in either English or French.

Course schedule: Tuesday and Thursday, 15h30 - 17h00, PK-4324. **Note:** Course meetings will be held over Zoom through 27 January.

Office hours: Office hours will be held by appointment, and usually over Zoom. Email me to schedule a meeting.

Course goals: Students will become familiar with the elementary theory of homological algebra.

Course description (from the course catalogue):

> modules; exact sequences; chain complexes and homology of chain complexes, simplicial and singular homology, cohomology, universal coefficient theorems, Tor and Ext functors, Künneth Formula, singular cohomology rings
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> modules; suites exactes; complexes de chaînes homologie d'un complexe de chaînes; homologie simpliciale et singulière; cohomologie; formule des coefficients universels; foncteurs Tor et Ext; formule de Künneth; anneau de cohomologie singulière

Prerequisites: Group theory and ring theory.

Primary text: J. J. Rotman, *An Introduction to Homological Algebra*, Second Edition, Universitext, Spinger, New York, 2009.

- Print and electronic versions are available from the library. https://uqam-bib.on.worldcat.org/oclc/430141797

Optional secondary texts

- C. A. Weibel, *An Introduction to Homological Algebra*, Cambridge Studies in Advanced Mathematics 38, Cambridge University Press, Cambridge, 1994.
  - Often considered the "gold standard", but a challenging book.
- S. Mac Lane, *Homology*, Classics in Mathematics 114, Springer, Berlin, 1995 reprint of 1975 edition.
  - This book is very readable, especially for those new to the subject, but the perspective is somewhat dated.
- S. Lang, *Algebra*, Revised Third Edition, Graduate Texts in Mathematics 211, Springer, New York, 2002.
  - This is a good reference for category theory and modules (Chapters I and III), and Part 4 (Chapters XX and XXI) offers a brief introduction to homological algebra.

Grading: We will have four problem sets due throughout the course, each worth 25% of the final grade. Solutions may be submitted in either English of French. Solutions will be graded on the following criteria:

- Correctness: Logical deductions and computations should be correct.
- Completeness: Every logical deduction should be justified. If a step is *actually* trivial, it is acceptable to say so as justification.
- Quality of mathematical writing: Proofs should be clear and easy to follow.

Problem sets will be submitted via email. If you would like to request an extension, please send me an email at least 24 hours before the deadline.

Academic integrity: Students are expected to follow the policies on academic integrity found in *Règlement no 18 sur les infractions de nature académique*. The full text of this regulation is available at https://instances.uqam.ca/reglements/.

Course Outline:

1. Modules
   (a) Definition and basic properties
   (b) Quivers and path algebras
   (c) Exact sequences
2. Category theory
   (a) Categories and functors
   (b) Natural transformations and the Yoneda Lemma
   (c) Exact functors
3. Special classes of modules
   (a) Tensor products
   (b) Projective modules
   (c) Flat modules
   (d) Injective modules
4. Complexes and homology
   (a) Chain complexes
   (b) Derived functors
5. Tor and Ext
   (a) Definitions and basic properties
   (b) Tor and torsion
   (c) Ext and extensions
   (d) Künneth Formula
   (e) Universal coefficient theorems

Disclaimer: Elements of this syllabus are subject to change at the instructor's discretion.